View metadata, citation and similar papers at core.ac.uk
brought to you by CORE
provided by Research Papers in Economics

Discussion Paper Series

No.127

A Further Extension of Duration Dependent Models

Satoru Kanoh

November 2005

Hitotsubashi University Research Unit
for Statistical Analysis in Social Sciences
A 21st-Century COE Program

Institute of Economic Research
Hitotsubashi University
Kunitachi, Tokyo, 186-8603 Japan
http://hi-stat.ier.hit-u.ac.jp/

A Further Extension of Duration Dependent Models

Abstract

The duration dependence of stock market cycles has been investigated using the Markov-switching model where the market conditions are unobservable. In the conventional modeling, restrictions are imposed that the transition probability is a monotonic function of duration and the duration is truncated at a certain value. This paper proposes a model that is free from these arbitrary restrictions and nests the conventional models. In the model, the parameters that characterize the transition probability are formulated in the state space. Empirical results in several stock markets show that the duration structures differ greatly depending on countries. They are not necessarily monotonic functions of duration and, therefore, cannot be described by the conventional models.

KEY WORDS: Duration, World stock markets, Markov-switching model, Nonparametric Model, Gibbs sampling, Marginal Likelihood

1 Introduction

In the analyses of business cycles and stock market cycles, duration dependence is an important issue that has attracted research interests. In the business cycle, the problem to be investigated is whether continuation of expansion or recession is dependent on how long the economy has been in those states. In the stock market, a similar question is asked about the continuation of bull or bear markets.

Duration dependence has been studied based on two different approaches. Cochrane and Defina (1995), Harman and Zuehlke (2004) and Zuehlke (2004) focus on the hazard rate of the change from one state to another. They formulated the hazard rates using Weibull distribution or modified versions. In this approach, a notable assumption is that the states are observable, which may be more suitable for the business cycle analysis than the stock market analysis, since the definition of the bull and the bear market is less clear than that of the expansion and contraction period. On the other hand, the Markov-switching model is first introduced by Hamilton (1989) where the switching mechanism between the states is expressed by a Markov model with constant transition probabilities. Diebold, Lee and Weinbach (1994) and Filardo (1994) extend the switching mechanism to the time-varying transition probabilities. Durland and McCurdy (1994) employ the Markov-switching model to analyze the U.S. business cycle where the transition probability is expressed as a function of duration. In this approach, the states are treated unobservable and estimated from data. Maheu and McCurdy (2000) use the same model for the U.S. stock market analysis.

This paper attempts to propose some extensions in the latter research stream. In Durland and McCurdy (1994), the transition probabilities are formulated as a logistic function of the duration at the last period and restrictions are imposed on the possible maximum length of duration in order to obtain stable estimates. As a result, the transition probabilities are monotonic functions of duration. Their estimation is based on the maximum likelihood. Pelagatti (2000) employs Bayesian MCMC methods to estimate the model in analyzing the U.S. business cycle. In the literature of the survival analysis, on the other hand, Garmerman (1992) proposes a non-parametric formulation of the hazard function. This paper combines the model with the Markov-switching model and creates a model that is exempted from the arbitrary restrictions including the monotonic dependency and nests the conventional models. In the model, the functional form of the transition probabilities is not specified explicitly. The transition probabilities are formulated in a state space model and they are recursively estimated using the Gibbs sampling and the Kalman filtering algorithm. Empirical results in several stock markets show that the actual duration dependence is not monotonic and, therefore, can be neither described by the conventional Markov-switching models nor by Weibull hazard models.

The remainder of this paper is organized as follows. In Section 2, the basic framework of the model and the estimation procedure are presented. In Section 3, applications of our model to the stock markets in various countries are presented. Section 4 states some concluding remarks.

2 Model Description

2.1 Formulation of the Model

The section starts with the description of the basic framework of the model. Consider the following model for a monthly stock return, R_t.

$$R_t = \mu_{S_t} + \epsilon_t, \quad \epsilon_t \sim N(0, \sigma^2_{S_t}), \tag{1}$$

$$\mu_{S_t} = \mu_0 + \mu_1 S_t, \quad \mu_0 > 0, \quad \mu_0 + \mu_1 < 0, \quad S_t = \{0, 1\}, \tag{2}$$

$$\sigma^2_{S_t} = (1 - S_t)\sigma^2_0 + S_t\sigma^2_1. \tag{3}$$

The state or regime S_t is an unobserved random variable that takes zero or one. The evolution of S_t is governed by a Markov process with the following,

$$\begin{cases} p_t^{(00)} := \Pr[S_t = 0 | S_{t-1} = 0, Z_{t-1}], \\ p_t^{(11)} := \Pr[S_t = 1 | S_{t-1} = 1, Z_{t-1}], \end{cases} \tag{4}$$

where Z_t is a vector of some variables. It holds that $p_t^{(01)} := \Pr[S_t = 0 | S_{t-1} = 1, Z_{t-1}] = 1 - p_t^{(11)}$, $p_t^{(10)} := \Pr[S_t = 1 | S_{t-1} = 0, Z_{t-1}] = 1 - p_t^{(00)}$.

The original Markov-switching model assumes that transition probabilities are constant. Diebold, Lee and Weinbach (1994) and Filardo (1994) extend them to time-varying probabilities. Such model is first applied to the U.S. stock market by Schaller and van Norder (1997). Maheu and McCurdy (2000) also use the duration of the market condition as an exogenous variable. Their model is based on the formulation by Durland and McCurdy (1994).

Let the duration be defined as:

$$D(S_t) = \begin{cases} D(S_{t-1}) + 1 & \text{if } S_t = S_{t-1} \\ 1 & \text{otherwise} \end{cases}$$

Note that the duration as well as the state S_t is unobservable variable. The following exemplifies the relationship between the state and corresponding duration value.

time(t)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	$\cdots$
S_t	1	1	1	1	1	0	0	0	1	1	0	0	0	0	$\cdots$
D_t	1	2	3	4	5	1	2	3	1	2	1	2	3	4	$\cdots$

In Durland and McCurdy (1994), the transition probabilities are expressed as :

$$P\left(S_t = i \,\middle|\, S_{t-1} = i, D(S_{t-1}) = d \right) = \frac{e^{\lambda_{i1} + \lambda_{i2} d}}{1 + e^{\lambda_{i1} + \lambda_{i2} d}}, \quad d \leq \tau, \quad i = 0, 1, \tag{5}$$

$$P\left(S_t = i \,\middle|\, S_{t-1} = i, D(S_{t-1}) = d \right) = \frac{e^{\lambda_{i1} + \lambda_{i2} \tau}}{1 + e^{\lambda_{i1} + \lambda_{i2} \tau}}, \quad d > \tau, \quad i = 0, 1, \tag{6}$$

The staying probability in the state i at time t and the hazard function has the following relationship:

$$h(t,i) = 1 - P\big(S_t = i | S_{t-1} = i, D(S_{t-1}) = d\big).$$

The formulation of Durland and McCurdy (1994) has the following characteristics. First, the transition probabilities that specify the switching mechanism are monotone functions of the duration. This excludes a more complicated duration dependence of transition probabilities. Second, in order to stabilize the estimation, a maximum limit of the duration period must be set. In an attempt to overcome these difficulties, this paper proposes a non-parametric model that does not assume the functional form of the transition probabilities and that nests the model by Durland and McCurdy (1994) as a special case. As we employ the Gibbs sampling estimation, it is more convenient to express the transition probabilities by probit functions instead of logistic functions. The transition probabities are expressed as :

$$\begin{cases} p_t^{(00)} = \Pr\{S_t^* < 0 | S_{t-1} = 0, D(S_{t-1}) = d\}, \\ p_t^{(11)} = \Pr\{S_t^* \geq 0 | S_{t-1} = 1, D(S_{t-1}) = d\}, \end{cases} \tag{7}$$

where

$$S_t^* = \varphi_{0,d}(1 - S_{t-1}) + \varphi_{1,d} S_{t-1} + u_t, \quad u_t \sim N(0,1), \tag{8}$$

$$\Delta^n \varphi_{i,d} = \eta_{i,d}, \quad \eta_{i,d} \sim N(0, \sigma^2_{\eta_{i,d}}), \quad (n \geq 1), \tag{9}$$

and S_t^* is a latent variable. (8) and (9) imply

$$X_{i,d} = \iota\, \varphi_{i,d} + \nu_{i,d}, \quad \nu_{i,d} \sim N(0, I),$$
$$\Delta^n \varphi_{i,d} = \eta_{i,d}, \quad \eta_{i,d} \sim N(0, \sigma^2_{\eta_{i,d}}),$$

for $i = 1, 2$ and $d = 1, 2, ..., \max(D(i))$. Here $\iota = [1, ..., 1]'$, I is a identity matrix. The empirical model in this paper considers the case when $n = 2$. Then, these are described as the following state space model.

$$\begin{bmatrix} x_{i,d}^{(1)} \\ x_{i,d}^{(2)} \\ \vdots \\ x_{i,d}^{(M_{i,d})} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ \vdots & \vdots \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \varphi_{i,d} \\ \varphi_{i,d-1} \end{bmatrix} + \begin{bmatrix} \nu_{i,d}^{(1)} \\ \nu_{i,d}^{(2)} \\ \vdots \\ \nu_{i,d}^{(M_{i,d})} \end{bmatrix}, \tag{10}$$

$$\begin{bmatrix} \varphi_{i,d} \\ \varphi_{i,d-1} \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \varphi_{i,d-1} \\ \varphi_{i,d-2} \end{bmatrix} + \begin{bmatrix} \eta_{i,d} \\ 0 \end{bmatrix}, \qquad (d \geq 2), \tag{11}$$

where $M_{i,d}$ is the dimension of observation vector.

In the measurement equation (10), the observation $x_{i,d}$ corresponds to S_t^*. $x_{i,d}$'s are obtained by sampling from a truncated normal distribution reflecting the information of state continuation or change. Note that the value of $M_{i,d}$ changes depending on d. The transition equation (11) describes the behavior of the parameter that specifies the transition probability with respect to the change of duration. When d is less than 2, the model is represented as $\iota = [1 \ \cdots \ 1]'$ and

$$\left[\ \varphi_{i,d}\ \right] = \left[\ 1\ \right]\left[\ \varphi_{i,d-1}\ \right] + \left[\ \eta_{i,d}\ \right], \qquad (d=1).$$

Here, $\sigma^2_{\eta_{i,d}}$ are assumed common for $d=1$ and for $d \geq 2$, i.e., $\sigma^2_{\eta_{i,d}} = \sigma^2_{\eta_i}$ for all d.

To conclude this subsection, the relationships of our model with the conventional Durland and McCurdy (1994) is briefly mentioned. If $\Delta^2\varphi_{i,d} = 0$ ($\sigma^2_{\eta_i} \approx 0$) for each i, φ is a linear function of d and the model reduces to Durland and McCurdy (1994). That is,

$$\begin{aligned} S_t^* &= \varphi_{0,d}(1-S_{t-1}) + \varphi_{1,d}S_{t-1} + u_t \\ &= (\gamma_0 + \gamma_1 d)(1-S_{t-1}) + (\lambda_0 + \lambda_1 d)S_{t-1} + u_t. \end{aligned}$$

Here

$$\begin{aligned} \gamma_0 &:= -\varphi_{0,2} + 2\varphi_{0,1}, \qquad & \lambda_0 &:= -\varphi_{1,2} + 2\varphi_{1,1}, \\ \gamma_1 &:= \varphi_{0,2} - \varphi_{0,1}, \qquad & \lambda_1 &:= \varphi_{1,2} - \varphi_{1,1}. \end{aligned}$$

2.2 Estimation

This section explains estimation of our model using the Gibbs sampling technique. Using the technique, the distributions of the parameters

$$\Theta = \{\underbrace{\mu_0, \mu_1}_{\mu}, \sigma_0^2, \sigma_1^2, \sigma_{\eta_0}^2, \sigma_{\eta_1}^2\} \tag{12}$$

and the latent variables

$$\Xi = \{\underbrace{S_1, ..., S_T}_{\tilde{S}_T}, \underbrace{S_1^*, ..., S_T^*}_{\tilde{S}_T^*}, \underbrace{\varphi_{0,1}, ..., \varphi_{0,\max(D(0))}}_{\tilde{\Phi}_{0,D}}, \underbrace{\varphi_{1,1}, ..., \varphi_{1,\max(D(1))}}_{\tilde{\Phi}_{1,D}}\} \tag{13}$$

are estimated.

In order to estimate Θ, the Gibbs sampling technique is employed and the following steps from

(a) to (f) are iterated. ($k = 1, 2, ..., M, M + 1, ..., M + N$)

(a) Generate $\mu^{(k+1)}$ from

$$g\left(\mu | \left\{\sigma_0^2\right\}^{(k)}, \left\{\sigma_1^2\right\}^{(k)}, \left\{\sigma_{\eta 0}^2\right\}^{(k)}, \left\{\sigma_{\eta_1}^2\right\}^{(k)}, \tilde{S}_T^{(k)}, \tilde{S}_T^{*(k)}, \tilde{\Phi}_{0,D}^{(k)}, \tilde{\Phi}_{1,D}^{(k)}\right),$$

(b) Generate $\left\{\sigma_0^2\right\}^{(k+1)}$ from

$$g\left(\sigma_0^2 \middle| \mu^{(k+1)}, \left\{\sigma_1^2\right\}^{(k)}, \left\{\sigma_{\eta 0}^2\right\}^{(k)}, \left\{\sigma_{\eta_1}^2\right\}^{(k)}, \tilde{S}_T^{(k)}, \tilde{S}_T^{*(k)}, \tilde{\Phi}_{0,D}^{(k)}, \tilde{\Phi}_{1,D}^{(k)}\right),$$

Generate $\left\{\sigma_1^2\right\}^{(k+1)}$ from

$$g\left(\sigma_1^2 \middle| \mu^{(k+1)}, \left\{\sigma_0^2\right\}^{(k+1)}, \left\{\sigma_{\eta 0}^2\right\}^{(k)}, \left\{\sigma_{\eta_1}^2\right\}^{(k)}, \tilde{S}_T^{(k)}, \tilde{S}_T^{*(k)}, \tilde{\Phi}_{0,D}^{(k)}, \tilde{\Phi}_{1,D}^{(k)}\right),$$

(c) Generate $\left\{\sigma_{\eta 0}^2\right\}^{(k+1)}$ from

$$g\left(\sigma_{\eta 0}^2 \middle| \mu^{(k+1)}, \left\{\sigma_0^2\right\}^{(k+1)}, \left\{\sigma_1^2\right\}^{(k+1)}, \left\{\sigma_{\eta_1}^2\right\}^{(k)}, \tilde{S}_T^{(k)}, \tilde{S}_T^{*(k)}, \tilde{\Phi}_{0,D}^{(k)}, \tilde{\Phi}_{1,D}^{(k)}\right),$$

Generate $\left\{\sigma_{\eta_1}^2\right\}^{(k+1)}$ from

$$g\left(\sigma_{\eta_1}^2 \middle| \mu^{(k+1)}, \left\{\sigma_0^2\right\}^{(k+1)}, \left\{\sigma_1^2\right\}^{(k+1)}, \left\{\sigma_{\eta 0}^2\right\}^{(k+1)}, \tilde{S}_T^{(k)}, \tilde{S}_T^{*(k)}, \tilde{\Phi}_{0,D}^{(k)}, \tilde{\Phi}_{1,D}^{(k)}\right),$$

(d) Generate $\tilde{S}_T^{(k+1)}$ from

$$g\left(\tilde{S}_T \middle| \mu^{(k+1)}, \left\{\sigma_0^2\right\}^{(k+1)}, \left\{\sigma_1^2\right\}^{(k+1)}, \left\{\sigma_{\eta 0}^2\right\}^{(k+1)}, \left\{\sigma_{\eta_1}^2\right\}^{(k+1)}, \tilde{S}_T^{*(k)}, \tilde{\Phi}_{0,D}^{(k)}, \tilde{\Phi}_{1,D}^{(k)}\right),$$

(e) Generate $\tilde{S}_T^{*(k+1)}$ from

$$g\left(\tilde{S}_T^* \middle| \mu^{(k+1)}, \left\{\sigma_0^2\right\}^{(k+1)}, \left\{\sigma_1^2\right\}^{(k+1)}, \left\{\sigma_{\eta 0}^2\right\}^{(k+1)}, \left\{\sigma_{\eta_1}^2\right\}^{(k+1)}, \tilde{S}_T^{(k+1)}, \tilde{\Phi}_{0,D}^{(k)}, \tilde{\Phi}_{1,D}^{(k)}\right),$$

(f) Generate $\tilde{\Phi}_{0,D}^{(k+1)}$ from

$$g\left(\tilde{\Phi}_{0,D} \middle| \mu^{(k+1)}, \left\{\sigma_0^2\right\}^{(k+1)}, \left\{\sigma_1^2\right\}^{(k+1)}, \left\{\sigma_{\eta 0}^2\right\}^{(k+1)}, \left\{\sigma_{\eta_1}^2\right\}^{(k+1)}, \tilde{S}_T^{(k+1)}, \tilde{S}_T^{*(k+1)}, \tilde{\Phi}_{1,D}^{(k)}\right),$$

Generate $\tilde{\Phi}_{1,D}^{(k+1)}$ from

$$g\left(\tilde{\Phi}_{1,D} \middle| \mu^{(k+1)}, \left\{\sigma_0^2\right\}^{(k+1)}, \left\{\sigma_1^2\right\}^{(k+1)}, \left\{\sigma_{\eta 0}^2\right\}^{(k+1)}, \left\{\sigma_{\eta_1}^2\right\}^{(k+1)}, \tilde{S}_T^{(k+1)}, \tilde{S}_{*T}^{(k+1)}, \tilde{\Phi}_{0,D}^{(k+1)}\right),$$

where g denotes an appropriate density function whose specification is explained below.

At the end of the $(M + N)$ th iteration, the first M results are dropped and the average values of the remaining N results are calculated as the estimates,

$$E[\Theta] = \frac{1}{N} \sum_{k=M+1}^{M+N} \Theta^{(k)}. \tag{14}$$

The generation of the estimates in steps (a) to (f) is summarized below.

First, in step (a), $\mu' = [\mu_0 \ \mu_1]$ is obtained from the posterior distribution (16) using the prior distribution (15).

$$\textbf{prior distribution}: \quad \mu|\sigma_0^2, \sigma_1^2, \tilde{R}_T, \tilde{S}_T^* \sim N(m_{0_\mu}, \Sigma_{0_\mu})_{I[\mu_0>\mu_1]}, \tag{15}$$

$$\textbf{posterior distribution}: \quad \mu|\sigma_0^2, \sigma_1^2, \tilde{R}_T, \tilde{S}_T^* \sim N(m_{1_\mu}, \Sigma_{1_\mu})_{I[\mu_0>\mu_1]}, \tag{16}$$

where

$$m_{1_\mu} = (\Sigma_{0_\mu}^{-1} + A_\mu' A_\mu)^{-1}(\Sigma_{0_\mu}^{-1} m_{0_\mu} + A_\mu' Y_\mu),$$

$$\Sigma_{1_\mu} = (\Sigma_{0_\mu}^{-1} + A_\mu' A_\mu)^{-1},$$

$$A_\mu' = \begin{bmatrix} 1/\sigma_{S_1} & \cdots & 1/\sigma_{S_T} \\ S_1/\sigma_{S_1} & \cdots & S_T/\sigma_{S_T} \end{bmatrix},$$

$$Y_\mu' = [R_1/\sigma_{S_1} \ \cdots \ R_T/\sigma_{S_T}],$$

and $I[\mu_0 > \mu_1]$ is an indicator function used to denote that the constraints $\mu_0 > \mu_1$ and $\mu_0 + \mu_1 < 0$ are satisfied.

In step (b), σ_0^2 and $\sigma_1^2 = (1+h)\sigma_0^2 = h^*\sigma_0^2$ are generated from the posterior distributions (19) and (20) using the prior densities (17) and (18), respectively.

$$\textbf{prior distribution}: \quad \sigma_0^2|\mu, h, \tilde{R}_T, \tilde{S}_T \sim IG\left(\frac{\nu_0}{2}, \frac{\delta_0}{2}\right), \tag{17}$$

$$h^*|\mu, \sigma_0^2, \tilde{R}_T, \tilde{S}_T \sim IG\left(\frac{\nu_{h0}}{2}, \frac{\delta_{h0}}{2}\right), \tag{18}$$

$$\textbf{posterior distribution}: \quad \sigma_0^2|\mu, h, \tilde{R}_T, \tilde{S}_T \sim IG\left(\frac{\nu_1}{2}, \frac{\delta_1}{2}\right), \tag{19}$$

$$h^*|\mu, \sigma_0^2, \tilde{R}_T, \tilde{S}_T \sim IG\left(\frac{\nu_{h1}}{2}, \frac{\delta_{h1}}{2}\right), \tag{20}$$

where

$$\nu_1 = \nu_0 + (T-p), \qquad \delta_1 = \delta_0 + \sum_{t=p+1}^{T} (\epsilon_t^*)^2,$$

$$\nu_{h1} = \nu_{h0} + T_h, \qquad \delta_{h1} = \delta_{h0} + \sum_{t=p+1}^{T} (\epsilon_t^{**})^2 S_t,$$

$$\epsilon_t^* = (R_t - \mu_{S_t})/\sqrt{1 + S_t h},$$

$$T_h = \sum_{t=p+1}^{T} S_t,$$

$$\epsilon_t^{**} = (R_t - \mu_{S_t})/\sigma_0.$$

In step (c), $\sigma_{\eta_0}^2$ and $\sigma_{\eta_1}^2$ are generated from the posterior distributions (23) and (24) using the prior densities (21) and (22), respectively.

$$\textbf{prior distribution}: \quad \sigma^2_{\eta_0}|\tilde{\Phi}_{0,D} \sim IG\left(\frac{\upsilon_{00}}{2}, \frac{\zeta_{00}}{2}\right), \tag{21}$$

$$\sigma^2_{\eta_1}|\tilde{\Phi}_{1,D} \sim IG\left(\frac{\upsilon_{10}}{2}, \frac{\zeta_{10}}{2}\right), \tag{22}$$

$$\textbf{posterior distribution}: \quad \sigma^2_{\eta_0}|\tilde{\Phi}_{0,D} \sim IG\left(\frac{\upsilon_{01}}{2}, \frac{\zeta_{01}}{2}\right), \tag{23}$$

$$\sigma^2_{\eta_1}|\tilde{\Phi}_{1,D} \sim IG\left(\frac{\upsilon_{11}}{2}, \frac{\zeta_{11}}{2}\right), \tag{24}$$

where

$$\upsilon_{01} = \upsilon_{00} + \max(D(0)), \quad \zeta_{01} = \zeta_{00} + Y_0'Y_0,$$

$$\upsilon_{11} = \upsilon_{10} + \max(D(1)), \quad \zeta_{11} = \zeta_{10} + Y_1'Y_1,$$

$$Y_0' = [\eta_{0,1} \ \cdots \ \eta_{0,\max(D(0))}], \ \eta_{0,d} = \varphi_{0,d} - 2\varphi_{0,d-1} + \varphi_{0,d-2},$$

$$Y_1 = [\eta_{1,1} \ \cdots \ \eta_{1,\max(D(1))}], \ \eta_{1,d} = \varphi_{1,d} - 2\varphi_{1,d-1} + \varphi_{1,d-2}.$$

Step (d) follows the method developed by Albert and Chib (1993).

$$P(S_t|\tilde{R}_T, S_{-t}) \propto P(S_t|S_{t-1})P(S_{t+1}|S_t)f(R_t|\tilde{R}_{t-1}, \tilde{S}_t), \tag{25}$$

where $\tilde{R}_t = \{R_1, ..., R_t\}$, $\tilde{S}_t = \{S_1, ..., S_t\}$, and $S_{-t} = \{S_1, ..., S_{t-1}, S_{t+1}, ..., S_T\}$.

In step (e), S_t^* is obtained by sampling u_t from a truncated normal distribution in such a way that S_t^* satisfies the inequalities given in (7).

In step (f), φ_t is obtained applying the algorithm by Carter and Korn (1994) that considered the Kalman Filtering estimation using the Gibbs sampling.

The comparison of the different models $(\mathcal{M}_1, ..., \mathcal{M}_I)$ is conducted based on the marginal likelihood following Chib (1995), i.e.,

$$\ln \hat{m}(\tilde{R}_T|\mathcal{M}_i) = \ln g(\tilde{R}_T|\Theta_i^*, \mathcal{M}_i) + \ln \pi(\Theta_i^*|\mathcal{M}_i) - \ln \hat{\pi}(\Theta_i^*|\tilde{R}_T, \mathcal{M}_i), \quad i = 1, 2, ..., I, \tag{26}$$

where Θ_i^* is the estimated parameter vector in the model $\mathcal{M}_i$, and $g(\tilde{R}_T|\Theta_i^*, \mathcal{M}_i)$, $\pi(\Theta_i^*|\mathcal{M}_i)$ and $\hat{\pi}(\Theta_i^*|\tilde{R}_T, \mathcal{M}_i)$ are the likelihood function, the prior density and the posterior density, respectively, for any model $i(\mathcal{M}_i)$. See Appendix for evaluation of the posterior density of the proposed model.

3 Duration Structure Analysis

3.1 Duration Structure Analysis of the U.S. Stock Market

Maheu and McCurdy (2000) estimate (5) and (6) in Durland and McCurdy(1994) using the monthly CRSP (Center for Research in Security Prices) Value Weighted Portfolio Return for NYSE from 1834:2 to 1995:12. They confirm that when the conditional mean has positive (negative) value, the volatility is relatively low (high). They call the market with positive conditional mean and relatively low volatility a bull market and, conversely, the market with negative conditional mean and relatively high volatility a bear market. Figure 1 plots the transition probabilities that are calculated using the estimation results in Maheu and McCurdy (2000). [1] The plots by □ and △ denote the staying probabilities in the bull market and the bear market, respectively. Both probabilities stay at constant levels after a certain period. This reflects the formulation of (5) and (6) where the upper limit of the duration (τ) was introduced. They set $\tau = 12$ that provides the maximum value of the likelihood function. From Figure 1, the staying probabilities increase with duration both in the bull market and bear market. The staying probability in the bull market exhibits high probability for small values of duration. On the other hand, the staying probability in the bear market is considerably low at the beginning. This implies that the probability is high that the bear market switches back to the bull market even immediately after a switch.

[Insert Figure 1 about here]

The estimation results of our model are as follows. The stock data used in our analysis is MSCI World Index (MSCI U.S., Local Currency) whose transition between 1970:1 and 2004:12 is depicted in Figure 2. Table 1 summarizes the relevant statistics of the logarithmic rate of return of the data.

[Insert Figure 2 about here]

[Insert Table 1 about here]

The parameters of the prior distributions in models are set as $m_{0_\mu} = [0.1, -0.2]$, $\Sigma_{0_\mu} = I$, $\nu_{00} = \nu_{10} = 1$, $\delta_0 = \delta_{h0} = 1$, $\upsilon_0 = \upsilon_{h0} = 1$, $\zeta_{00} = \zeta_{10} = 1$. The dropped draws M and the remaining draws N are 3,000 and 7,000, respectively.

Table 2 shows the statistics concerning the distribution of the estimated parameters. From Table 2, the states of the U.S. market can be classified into the market with a positive mean and a smaller variance and that with a negative mean and a larger variance. This classification is consistent with that by Maheu and McCurdy (2000). Table 2 also shows that the value of the marginal likelihood of our model is greater than that of the model by Durland and McCurdy (1994).

[Insert Table 2 about here]

The upper-left panel of Figure 3 illustrates the transition of $\varphi_{i,d}$ according to the change of duration that are calculated as the average of the 7,000 iterations. Similarly, the upper-right panel illustrates the transition of staying probabilities that are calculated as the average of 7000 staying probabilities each of which is transformed from $\varphi_{i,d}$ obtained at each iteration. In these figures, the duration length is truncated at 96 months (8 years). As in Figure 1, the plot of $\Box$ shows the values for the bull market and the plot by $\triangle$ shows those for the bear market. These symbols are plotted only for range less than 95 percentile points of the distributions among the 7,000 iterations of max(D(0)) and max(D(1)). Outside of this range, the staying values are shown simply by lines. When the duration characteristics are closely watched within 95 percent range, the following points are noticed. First, $\varphi_{1,d}$ of the bear market increases more and less linearly, with duration. This result is compatible with that by Maheu and McCurdy (2000). The characteristic of the bull market is, however, that $\varphi_{0,d}$ does not change greatly and stays generally at the same level from the beginning. Second, the maximum duration of the bull market is approximately ten times as long as that of the bear market. This difference is not considered in Maheu and McCurdy (2000) and could be the factor leading to the improvement of the marginal likelihood.

[Insert Figure 3 about here]

3.2 Application to the Other Stock Markets

In this subsection, the duration structures of the markets in six countries (U.K., France, Germany, Japan, Singapore, Hong Kong) other than U.S. are investigated using the same model. As before, the MSCI World Indices (Local Currency) are used as the indicators of these markets. The movements of these indices are dipicted in Figures 4 to 9 and the characteristics of them are summarized in Table 3. The markets in U.K., France and Germany show similar behaviors with the U.S. market. Remaining markets, i.e. Japan, Singapore, Hong Kong markets exhibit different behaviors individually.

[Insert Figures 4 to 9 about here]

[Insert Table 3 about here]

In the estimation, the same prior densities with the U.S. market are employed. Tables 4 to 9 show the statistics concerning the distribution of the estimated parameters. The mean and variance characteristics regarding the bull and bear markets in the six countries have the common feature with the U.S. market. That is, the bull market is characterized by a positive mean and a smaller variance and the bear market is characterized by a negative mean and a larger variance. In any countries, the values of the marginal likelihood of our model exceed those of the model by Durland and McCurdy (1994).

[Insert Tables 4 to 9 about here]

By looking at the upper-left panels of figures 10 to 15, let us investigate the relationships between duration and $\varphi_{i,d}\,(i=0,1)$ in these countries. First, France and Germany show similar characteristics with U.S. That is, in the bear market, $\varphi_{1,d}$ increases more or less linearly. On the other hand, in the bull market, $\varphi_{0,d}$ stays generally at the same level with duration. Second, the characteristics of the bull market in U.K. and Hong Kong are similar to those in U.S., France and Germany. However, the characteristics of the bear market are different. Note that $\varphi_{1,d}$ in U.K. increases until approximately one year and decreases then, while $\varphi_{1,d}$ in Hong Kong increases until approximately two years and decreases afterwards. Third, Japanese characteristics of duration are very different from the others. That is, both $\varphi_{0,d}$ and $\varphi_{1,d}$ are approximately linear in d. The other conspicuous points observed in the upper-right panel of figure 13 are that the duration period in the bull market is short and that the staying probability in the bear market stays above the staying probability in the bull market. Finally, in Singapore, $\varphi_{1,d}$ of the bear market shows ups and downs with upward trend, while $\varphi_{0,d}$ of the bull market decreases until approximately one year and half and then starts increasing.

To summarize, these empirical results suggest that the duration structures differ greatly depending on countries and are not necessarily monotonic.

[Insert Figures 10 to 15 about here]

4 Conclusion

This paper proposed some extensions in the analysis of duration within the framework of the Markov-switching models. It constructed a model that was exempted from arbitrary restrictions of conventional models and nested them. In the model, the transition probabilities were formulated in the state space and were recursively estimated using the Gibbs sampling and Kalman Filter algorithm.

The empirical analyses showed that the mean and variance characteristics of the bull and bear markets in various countries have common features. First, the bull market is characterized by a positive mean and a smaller variance and the bear market is characterized by a negative mean and a larger variance. Secondly, the duration period in the bear market is shorter than in the bull market.

Country by country analysis revealed, however, the following differences in the relationships between durations and staying probabilities among the countries. First, U.S., France and Germany exhibited similar characteristics. That is, in the bear market, the staying probabilities increased, more and less, with duration. In the bull market, the staying probabilities stayed by and large at the same level with duration. The characteristics of the bull market in U.K. and Hong Kong were similar to those in U.S., France and Germany. However, the characteristics of the bear market were different. The staying probability increased until approximately one year and decreased then, while the staying probability in Hong Kong increases until approximately two years and decreases afterwards. Japanese characteristics of duration were very different from the others. The conspicuous points were that the duration period in the bull market was short and that the staying probability in the bear market was larger than the staying probability in the bull market. Both staying probabilities were monotone increasing function of duration. Finally, in Singapore, the staying probability in the bear market tended to increase generally but the probability in the bull market started falling after one year and half. To summarize our empirical results, the duration structures differ greatly depending on countries. The transition probabilities are not necessarily monotonic function of duration and, therefore, could not be described by the conventional models.

A future topic to be investigated may be the time dependency of the duration structure. The transition probability in this paper changes with time only through the change of duration. It takes the same value for a given value of duration even at different time. It would be interesting to extend the duration structure to vary from time to time as the market conditions change.

Footnotes

[1] Maheu and McCurdy (2000) study four types of AR(5) models including Durland and McCurdy(1994) for the return process. They report that the best model is the one where the conditional means and variances are different for the two states and the duration is an explanatory variable for the conditional means and variances.

Appendix: Evaluation of the Posterior Density of the Proposed Model

The rules of probability imply that

$$\begin{aligned}
\pi\left(\Theta^*\middle|\,\tilde{R}_T\right) &= \pi\left(\mu^*, \left\{\sigma_0^2\right\}^*, \left\{\sigma_1^2\right\}^*, \left\{\sigma_{\eta_0}^2\right\}^*, \left\{\sigma_{\eta_1}^2\right\}^* \middle|\, \tilde{R}_T\right) \\
&= \pi\left(\left\{\sigma_{\eta_0}^2\right\}^* \middle|\, \tilde{R}_T\right) \times \pi\left(\left\{\sigma_{\eta_1}^2\right\}^* \middle|\, \tilde{R}_T, \left\{\sigma_{\eta_0}^2\right\}^*\right) \\
&\quad \times \pi\left(\left\{\sigma_0^2\right\}^* \middle|\, \tilde{R}_T, \left\{\sigma_{\eta_0}^2\right\}^*, \left\{\sigma_{\eta_1}^2\right\}^*\right) \times \pi\left(\left\{\sigma_1^2\right\}^* \middle|\, \tilde{R}_T, \left\{\sigma_{\eta_0}^2\right\}^*, \left\{\sigma_{\eta_1}^2\right\}^*, \left\{\sigma_0^2\right\}^*\right) \\
&\quad \times \pi\left(\mu^*\middle|\, \tilde{R}_T, \left\{\sigma_{\eta_0}^2\right\}^*, \left\{\sigma_{\eta_1}^2\right\}^*, \left\{\sigma_0^2\right\}^*, \left\{\sigma_1^2\right\}^*\right), \qquad (27)
\end{aligned}$$

where

$$\begin{aligned}
&\pi\left(\left\{\sigma_{\eta_0}^2\right\}^* \middle|\, \tilde{R}_T\right) \\
&= \int\int\int\int\int\int\int \pi\left(\left\{\sigma_{\eta_0}^2\right\}^* \middle|\, \tilde{R}_T, \sigma_{\eta_1}^2, \sigma_0^2, \sigma_1^2, \mu, \tilde{S}_T, \tilde{\Phi}_{0,D}, \tilde{\Phi}_{1,D}\right) \\
&\quad \times \pi\left(\sigma_{\eta_1}^2, \sigma_0^2, \sigma_1^2, \mu, \tilde{S}_T, \tilde{\Phi}_{0,D}, \tilde{\Phi}_{1,D} \middle|\, \tilde{R}_T\right) d\sigma_{\eta_0}^2 d\sigma_0^2 d\sigma_1^2 d\mu d\tilde{S}_T d\tilde{\Phi}_{0,D} d\tilde{\Phi}_{1,D}, \qquad (28) \\
&\pi\left(\left\{\sigma_{\eta_1}^2\right\}^* \middle|\, \tilde{R}_T, \left\{\sigma_{\eta_0}^2\right\}^*\right) \\
&= \int\int\int\int\int\int \pi\left(\left\{\sigma_{\eta_1}^2\right\}^* \middle|\, \tilde{R}_T, \left\{\sigma_{\eta_0}^2\right\}^*, \sigma_0^2, \sigma_1^2, \mu, \tilde{S}_T, \tilde{\Phi}_{0,D}, \tilde{\Phi}_{1,D}\right) \\
&\quad \times \pi\left(\sigma_0^2, \sigma_1^2, \mu, \tilde{S}_T, \tilde{\Phi}_{0,D}, \tilde{\Phi}_{1,D} \middle|\, \tilde{R}_T\right) d\sigma_0^2 d\sigma_1^2 d\mu d\tilde{S}_T d\tilde{\Phi}_{0,D} d\tilde{\Phi}_{1,D}, \qquad (29) \\
&\pi\left(\left\{\sigma_0^2\right\}^* \middle|\, \tilde{R}_T, \left\{\sigma_{\eta_0}^2\right\}^*, \left\{\sigma_{\eta_1}^2\right\}^*\right) \\
&= \int\int\int\int\int \pi\left(\left\{\sigma_0^2\right\}^* \middle|\, \tilde{R}_T, \left\{\sigma_{\eta_0}^2\right\}^*, \left\{\sigma_{\eta_1}^2\right\}^*, \sigma_1^2, \mu, \tilde{S}_T, \tilde{\Phi}_{0,D}, \tilde{\Phi}_{1,D}\right) \\
&\quad \times \pi\left(\sigma_1^2, \mu, \tilde{S}_T, \tilde{\Phi}_{0,D}, \tilde{\Phi}_{1,D} \middle|\, \tilde{R}_T\right) d\sigma_1^2 d\mu d\tilde{S}_T d\tilde{\Phi}_{0,D} d\tilde{\Phi}_{1,D}, \qquad (30) \\
&\pi\left(\left\{\sigma_1^2\right\}^* \middle|\, \tilde{R}_T, \left\{\sigma_{\eta_0}^2\right\}^*, \left\{\sigma_{\eta_1}^2\right\}^*, \left\{\sigma_0^2\right\}^*\right) \\
&= \int\int\int\int \pi\left(\left\{\sigma_1^2\right\}^* \middle|\, \tilde{R}_T, \left\{\sigma_{\eta_0}^2\right\}^*, \left\{\sigma_{\eta_1}^2\right\}^*, \left\{\sigma_0^2\right\}^*, \mu, \tilde{S}_T, \tilde{\Phi}_{0,D}, \tilde{\Phi}_{1,D}\right) \\
&\quad \times \pi\left(\mu, \tilde{S}_T, \tilde{\Phi}_{0,D}, \tilde{\Phi}_{1,D} \middle|\, \tilde{R}_T\right) d\mu d\tilde{S}_T d\tilde{\Phi}_{0,D} d\tilde{\Phi}_{1,D}, \qquad (31) \\
&\pi\left(\mu^*\middle|\, \tilde{R}_T, \left\{\sigma_{\eta_0}^2\right\}^*, \left\{\sigma_{\eta_1}^2\right\}^*, \left\{\sigma_0^2\right\}^*, \left\{\sigma_1^2\right\}^*\right) \\
&= \int\int\int \pi\left(\mu^*\middle|\, \tilde{R}_T, \left\{\sigma_{\eta_0}^2\right\}^*, \left\{\sigma_{\eta_1}^2\right\}^*, \left\{\sigma_0^2\right\}^*, \left\{\sigma_1^2\right\}^*, \tilde{S}_T, \tilde{\Phi}_{0,D}, \tilde{\Phi}_{1,D}\right) \\
&\quad \times \pi\left(\tilde{S}_T, \tilde{\Phi}_{0,D}, \tilde{\Phi}_{1,D} \middle|\, \tilde{R}_T\right) d\mu d\tilde{S}_T d\tilde{\Phi}_{0,D} d\tilde{\Phi}_{1,D}. \qquad (32)
\end{aligned}$$

Using the Gibbs sampler outputs, (28) - (32) and the week law of large numbers imply that

$$\begin{aligned}
&\hat{\pi}\left(\left\{\sigma_{\eta_0}^2\right\}^*\middle|\tilde{R}_T\right)\\
&=\frac{1}{G}\sum_{g=1}^{G}\pi\left(\left\{\sigma_{\eta_0}^2\right\}^*\middle|\tilde{R}_T,\left\{\sigma_{\eta_1}^2\right\}^{(g)},\left\{\sigma_0^2\right\}^{(g)},\left\{\sigma_1^2\right\}^{(g)},\mu^{(g)},\tilde{S}_T^{(g)},\tilde{\Phi}_{0,D}^{(g)},\tilde{\Phi}_{1,D}^{(g)}\right)\\
&=\frac{1}{G}\sum_{g=1}^{G}\pi\left(\left\{\sigma_{\eta_0}^2\right\}^*\middle|\tilde{\Phi}_{0,D}^{(g)}\right), \qquad (33)\\
&\hat{\pi}\left(\left\{\sigma_{\eta_1}^2\right\}^*\middle|\tilde{R}_T,\left\{\sigma_{\eta_0}^2\right\}^*\right)\\
&=\frac{1}{G}\sum_{g=1}^{G}\pi\left(\left\{\sigma_{\eta_1}^2\right\}^*\middle|\tilde{R}_T,\left\{\sigma_{\eta_0}^2\right\}^*,\left\{\sigma_0^2\right\}^{(g)},\left\{\sigma_1^2\right\}^{(g)},\mu^{(g)},\tilde{S}_T^{(g)},\tilde{\Phi}_{0,D}^{(g)},\tilde{\Phi}_{1,D}^{(g)}\right)\\
&=\frac{1}{G}\sum_{g=1}^{G}\pi\left(\left\{\sigma_{\eta_1}^2\right\}^*\middle|\tilde{\Phi}_{1,D}^{(g)}\right), \qquad (34)\\
&\hat{\pi}\left(\left\{\sigma_0^2\right\}^*\middle|\tilde{R}_T,\left\{\sigma_{\eta_0}^2\right\}^*,\left\{\sigma_{\eta_1}^2\right\}^*\right)\\
&=\frac{1}{G}\sum_{g=1}^{G}\pi\left(\left\{\sigma_0^2\right\}^*\middle|\tilde{R}_T,\left\{\sigma_{\eta_0}^2\right\}^*,\left\{\sigma_{\eta_1}^2\right\}^*,\left\{\sigma_1^2\right\}^{(g)},\mu^{(g)},\tilde{S}_T^{(g)},\tilde{\Phi}_{0,D}^{(g)},\tilde{\Phi}_{1,D}^{(g)}\right)\\
&=\frac{1}{G}\sum_{g=1}^{G}\pi\left(\left\{\sigma_0^2\right\}^*\middle|\tilde{R}_T,\left\{\sigma_1^2\right\}^{(g)},\mu^{(g)},\tilde{S}_T^{(g)}\right), \qquad (35)\\
&\hat{\pi}\left(\left\{\sigma_1^2\right\}^*\middle|\tilde{R}_T,\left\{\sigma_{\eta_0}^2\right\}^*,\left\{\sigma_{\eta_1}^2\right\}^*,\left\{\sigma_0^2\right\}^*\right)\\
&=\frac{1}{G}\sum_{g=1}^{G}\pi\left(\left\{\sigma_1^2\right\}^*\middle|\tilde{R}_T,\left\{\sigma_{\eta_0}^2\right\}^*,\left\{\sigma_{\eta_1}^2\right\}^*,\left\{\sigma_0^2\right\}^*,\mu^{(g)},\tilde{S}_T^{(g)},\tilde{\Phi}_{0,D}^{(g)},\tilde{\Phi}_{1,D}^{(g)}\right)\\
&=\frac{1}{G}\sum_{g=1}^{G}\pi\left(\left\{\sigma_1^2\right\}^*\middle|\tilde{R}_T,\left\{\sigma_0^2\right\}^*,\mu^{(g)},\tilde{S}_T^{(g)}\right), \qquad (36)\\
&\hat{\pi}\left(\mu^*\middle|\tilde{R}_T,\left\{\sigma_{\eta_0}^2\right\}^*,\left\{\sigma_{\eta_1}^2\right\}^*,\left\{\sigma_0^2\right\}^*,\left\{\sigma_1^2\right\}^*\right)\\
&=\frac{1}{G}\sum_{g=1}^{G}\pi\left(\mu^*\middle|\tilde{R}_T,\left\{\sigma_{\eta_0}^2\right\}^*,\left\{\sigma_{\eta_1}^2\right\}^*,\left\{\sigma_0^2\right\}^*,\left\{\sigma_1^2\right\}^*,\tilde{S}_T^{(g)},\tilde{\Phi}_{0,D}^{(g)},\tilde{\Phi}_{1,D}^{(g)}\right)\\
&=\frac{1}{G}\sum_{g=1}^{G}\pi\left(\mu^*\middle|\tilde{R}_T,\left\{\sigma_0^2\right\}^*,\left\{\sigma_1^2\right\}^*,\tilde{S}_T^{(g)}\right). \qquad (37)
\end{aligned}$$

Finally, we obtain the third term of (26) by calculating the following equation:

$$
\begin{aligned}
\ln \hat{\pi}(\Theta_i^* | \tilde{R}_T) \quad = \quad & \ln \left[\frac{1}{G} \sum_{g=1}^{G} \pi \left(\left\{ \sigma_{\eta_0}^2 \right\}^* \middle| \tilde{\Phi}_{0,D}^{(g)} \right) \right] \\
& + \ln \left[\frac{1}{G} \sum_{g=1}^{G} \pi \left(\left\{ \sigma_{\eta_1}^2 \right\}^* \middle| \tilde{\Phi}_{1,D}^{(g)} \right) \right] \\
& + \ln \left[\frac{1}{G} \sum_{g=1}^{G} \pi \left(\left\{ \sigma_0^2 \right\}^* \middle| \tilde{R}_T, \left\{ \sigma_1^2 \right\}^{(g)}, \mu^{(g)}, \tilde{S}_T^{(g)} \right) \right] \\
& + \ln \left[\frac{1}{G} \sum_{g=1}^{G} \pi \left(\left\{ \sigma_1^2 \right\}^* \middle| \tilde{R}_T, \left\{ \sigma_0^2 \right\}^*, \mu^{(g)}, \tilde{S}_T^{(g)} \right) \right] \\
& + \ln \left[\frac{1}{G} \sum_{g=1}^{G} \pi \left(\Phi^* | \tilde{R}_T, \left\{ \sigma_0^2 \right\}^*, \left\{ \sigma_1^2 \right\}^*, \mu^{(g)}, \tilde{S}_T^{(g)} \right) \right] \\
& + \ln \left[\frac{1}{G} \sum_{g=1}^{G} \pi \left(\mu^* | \tilde{R}_T, \left\{ \sigma_0^2 \right\}^*, \left\{ \sigma_1^2 \right\}^*, \tilde{S}_T^{(g)} \right) \right]. \qquad (38)
\end{aligned}
$$

References

Albert, J. H., and Chib, S. (1993), Bayes Inference via Gibbs Sampling of Autoregressive Time Series Subject to Markov Mean and Variance Shifts, *Journal of Business & Economic Statistics*, 11, 1 - 15.

Carter, C. K., and Korn, R. (1994), On Gibbs Sampling for State Space Models, *Biometrika*, 81, 541 - 553.

Chib, S. (1995), Marginal Likelihood from the Gibbs Output, *Journal of the American Statistical Association*, 90, 1313 - 1321.

Cochrance, S. J., and Defina, R. H. (1995), Duration dependence in the US stock market cycle: a parametric approach, *Applied Finacial Economics*, 5, 309-318.

Diebold, F. X., Lee, J., H., and Weinbach, G. C. (1994), Regime Switching with Time-varying Transition Probabilities, in *Nonstationary Time Series Analysis and Cointegration*, ed. C. Hargreaves, Oxford University Press, Oxford.

Durland, J. M., and McCurdy, T. H. (1994), Duration-Dependent Transitions in a Markov Model of U.S. GNP Growth, *Journal of Business & Economic Statistics*, 12, 279 - 288.

Filardo, A. J. (1994), Business Cycle Phases and Their Transition Dynamics, *Journal of Business & Economic Statistics*, 12, 299 - 308.

Gamerman, D. (1992), A Dynamic Approach to Statistical Analysis of processes, *Biometrika*, 79, 39 - 50.

Hamilton, J. D. (1989), A New Approach to the Economic Analysis of Non-stationary Time Series and the Business Cycle, *Econometrica*, 57, 357 - 384.

Harman, Y., and Zuehlke, T. W. (2004), Duration Dependence in Stock Market Cycles Reconsidered, Working paper.

Maheu, M., and McCurdy, T. H. (2000), Identifying Bull and Bear Markets in Stock Returns, *Journal of Business & Economic Statistics*, 18, 100 - 112.

Pelagatti, M. (2000), Gibbs Sampling for a Duration Dependent Markov Switching Model with an Application to the U.S. Business Cycle, Quaderno di Dipartimento QD2001/2, Dipartimento di Statistica, Universita' degli Studi di Milano Bicocca.

Schaller, H., and van Norden, S. (1997), Regime Switching in Stock Market Returns, *Applied Financial Economics*, 7, 177 - 191.

Zuehlke, T. W. (2004), Business Cycle Duration Dependence Reconsidered, *Journal of Business & Economic Statistics*, 21.

Table 1: Summary Statistics of U.S. Stock Return (Local Currency) 1970:1-2004:12

average	stdev.	min.	25%	median	75%	max.
0.5958	6.0214	-24.8101	-3.3831	1.1470	4.7600	20.3974

Table 2: Summary of Estimation in U.S. (1970:1-2004:12)

	μ_0	μ_1	σ_0^2	σ_1^2	$\sigma_{\eta_0}^2$	$\sigma_{\eta_1}^2$	Max(D(0))	Max(D(1))
average	0.218	-0.655	0.736	3.247	0.832	2.592	72.8	7.6
stdev	0.056	0.340	0.093	1.586	0.417	34.287	38.3	3.4
median	0.216	-0.586	0.737	2.822	0.762	0.808	63	7
5%	0.129	-1.297	0.583	1.768	0.291	0.204	26	4
95%	0.310	-0.241	0.888	6.232	1.608	5.743	151	14

		Durland and McCurdy's model	
	our model	$\tau = 12$	$\tau = 24$
marginal likelihood	-553.00	-559.08	-562.60

Table 3: Summary Statistics of Some MSCI Word Index Return (Local Currency) 1970:1-2004:12

	average	stdev.	min.	25%	median	75%	max.
U.K.	0.6409	5.8549	-30.2756	-2.1829	1.0210	3.7458	42.8879
France	0.5958	6.0214	-24.8101	-3.3831	1.1470	4.7600	20.3974
Germany	0.4067	5.7930	-28.6742	-2.6174	0.6868	4.1130	17.9559
Japan	0.4686	5.3152	-21.8722	-2.7366	0.6156	3.5582	18.2042
Singapore	0.5857	8.2036	-54.3873	-2.7325	0.6449	4.5301	38.5884
Hong Kong	1.0237	10.3932	-57.4133	-3.6134	1.1538	6.7340	54.1685

Table 4: Summary of Estimation in U.K. (1970:1-2004:12)

	μ_0	μ_1	σ_0^2	σ_1^2	$\sigma_{\eta_0}^2$	$\sigma_{\eta_1}^2$	Max(D(0))	Max(D(1))
average	0.155	-0.528	0.579	6.985	1.413	3.299	139.8	17.1
stdev	0.041	0.299	0.054	2.915	0.536	4.021	45.2	4.0
median	0.155	-0.465	0.576	6.390	1.354	2.237	142	18
5%	0.087	-1.106	0.494	3.820	0.653	0.550	70	9
95%	0.223	-0.174	0.671	11.882	2.396	9.514	204	23

		Durland and McCurdy's model	
	our model	$\tau = 12$	$\tau = 24$
marginal likelihood	-512.95	-517.26	-521.75

Table 5: Summary of Estimation in France (1970:1-2004:12)

	μ_0	μ_1	σ_0^2	σ_1^2	$\sigma_{\eta_0}^2$	$\sigma_{\eta_1}^2$	Max(D(0))	Max(D(1))
average	0.206	-0.460	0.702	2.293	0.659	1.411	45.6	10.1
stdev	0.064	0.246	0.097	0.809	0.374	7.974	27.0	5.3
median	0.203	-0.404	0.700	2.110	0.578	0.671	39	9
5%	0.108	-0.922	0.547	1.414	0.218	0.191	17	4
95%	0.316	-0.180	0.867	3.817	1.375	3.632	93	21

		Durland and McCurdy's model	
	our model	$\tau = 12$	$\tau = 24$
marginal likelihood	-560.28	-565.63	-565.04

Table 6: Summary of Estimation in Germany (1970:1-2004:12)

	μ_0	μ_1	σ_0^2	σ_1^2	$\sigma_{\eta_0}^2$	$\sigma_{\eta_1}^2$	Max(D(0))	Max(D(1))
average	0.161	-0.385	0.556	2.727	0.858	1.106	60.7	14.1
stdev	0.050	0.210	0.083	0.750	0.442	1.650	31.0	7.0
median	0.160	-0.336	0.553	2.572	0.769	0.749	53	13
5%	0.078	-0.794	0.425	1.829	0.308	0.212	25	6
95%	0.242	-0.137	0.698	4.162	1.690	3.003	121	27

		Durland and McCurdy's model	
	our model	$\tau = 12$	$\tau = 24$
marginal likelihood	-532.84	-542.00	-542.61

Table 7: Summary of Estimation in Japan (1970:1-2004:12)

	μ_0	μ_1	σ_0^2	σ_1^2	$\sigma_{\eta_0}^2$	$\sigma_{\eta_1}^2$	Max(D(0))	Max(D(1))
average	0.186	-0.266	0.467	1.752	0.581	0.623	14.9	10.1
stdev	0.071	0.106	0.102	0.299	0.390	0.423	8.1	4.6
median	0.182	-0.255	0.462	1.712	0.468	0.498	10	9
5%	0.075	-0.460	0.305	1.339	0.157	0.155	6	6
95%	0.306	-0.110	0.644	2.311	1.423	1.530	30	18

		Durland and McCurdy's model	
	our model	$\tau = 12$	$\tau = 24$
marginal likelihood	-539.86	-545.52	-549.28

Table 8: Summary of Estimation in Hong Kong (1970:1-2004:12)

	μ_0	μ_1	σ_0^2	σ_1^2	$\sigma_{\eta_0}^2$	$\sigma_{\eta_1}^2$	Max(D(0))	Max(D(1))
average	0.173	-0.305	0.440	3.126	0.605	1.368	38.0	14.6
stdev	0.046	0.129	0.064	0.755	0.338	3.097	13.0	8.6
median	0.173	-0.279	0.438	3.004	0.535	0.861	37	12
5%	0.099	-0.546	0.339	2.147	0.215	0.230	19	5
95%	0.250	-0.144	0.550	4.535	1.227	3.699	66	31

		Durland and McCurdy's model	
	our model	$\tau = 12$	$\tau = 24$
marginal likelihood	-500.43	-510.01	-517.08

Table 9: Summary of Estimation in Singapore (1970:1-2004:12)

	μ_0	μ_1	σ_0^2	σ_1^2	$\sigma_{\eta_0}^2$	$\sigma_{\eta_1}^2$	Max(D(0))	Max(D(1))
average	0.117	-0.211	0.313	2.803	0.581	0.916	36.9	19.3
stdev	0.039	0.095	0.042	0.520	0.313	0.762	7.1	8.1
median	0.117	-0.194	0.311	2.733	0.512	0.717	37	17
5%	0.053	-0.391	0.248	2.091	0.223	0.218	25	9
95%	0.180	-0.090	0.386	3.754	1.178	2.291	47	35

		Durland and McCurdy's model	
	our model	$\tau = 12$	$\tau = 24$
marginal likelihood	-471.86	-479.87	-497.85

Figure 1: Duration Structure of the U.S. Stock Market Using Maheu and McCudy (2000) Results

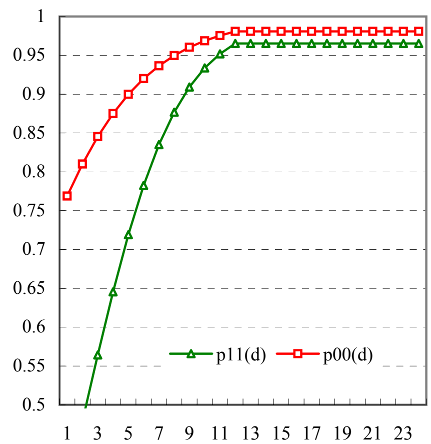

Figure 2: U.S. Stock Index (Local Currency) 1970:1-2004:12

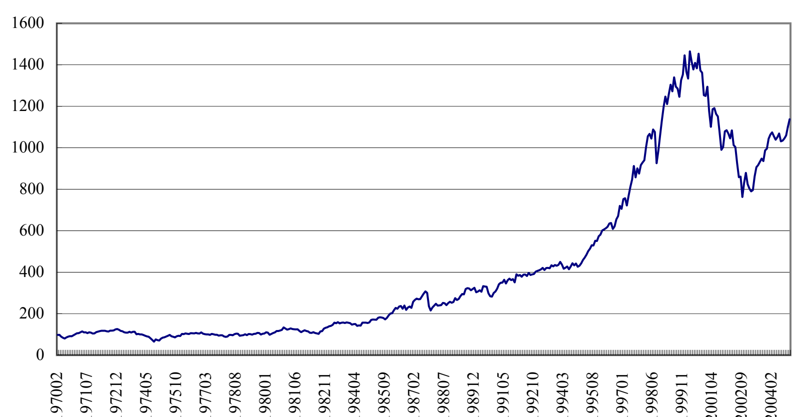

Figure 3: Bull-Bear and Duration Structure in U.S. (1970:1-2004:12)

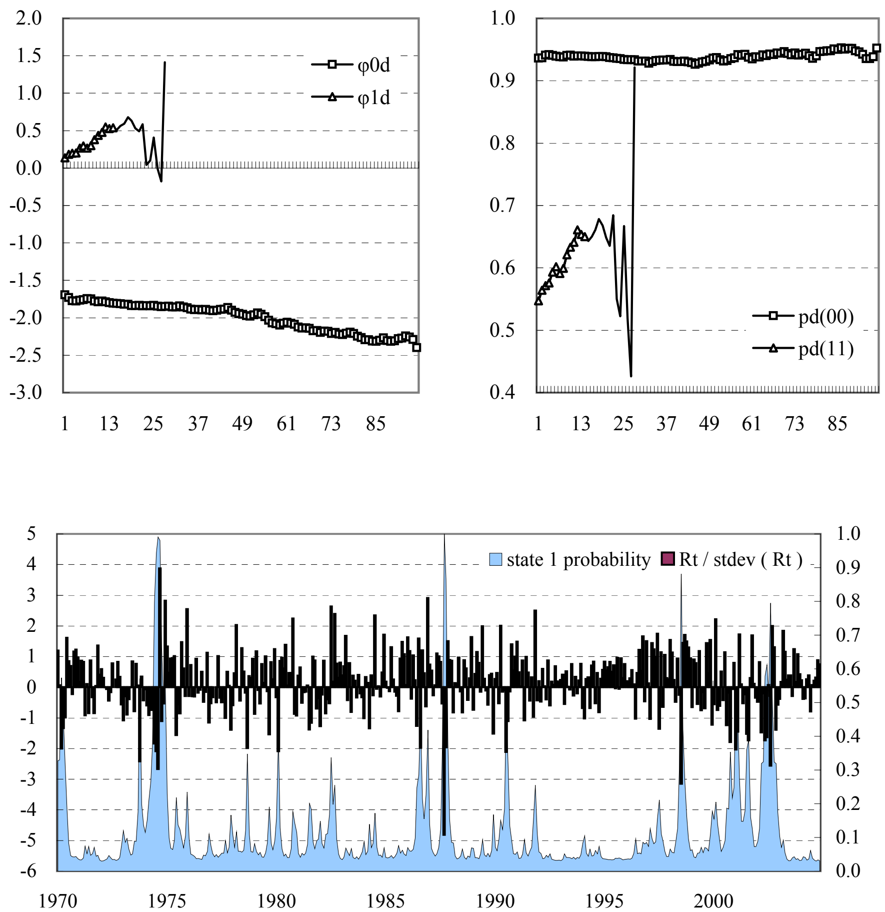

Figure 4: U.K.

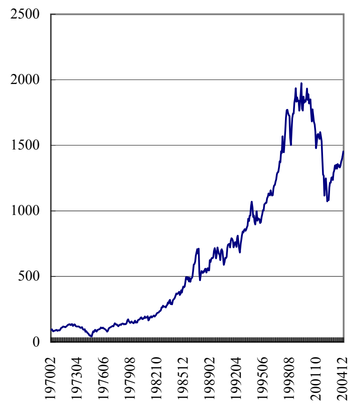

Figure 5: France

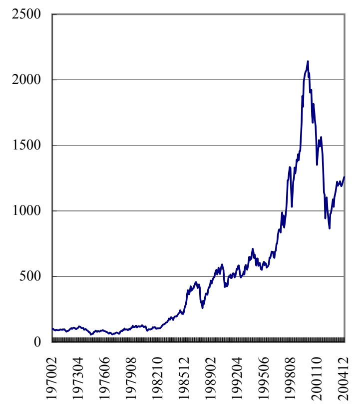

Figure 6: Germany

Figure 7: Japan

Figure 8: Singapore

Figure 9: Hong Kong

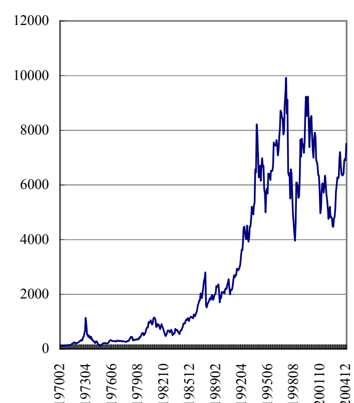

Figure 10: Bull-Bear and Duration Structure in U.K. (1970:1-2004:12)

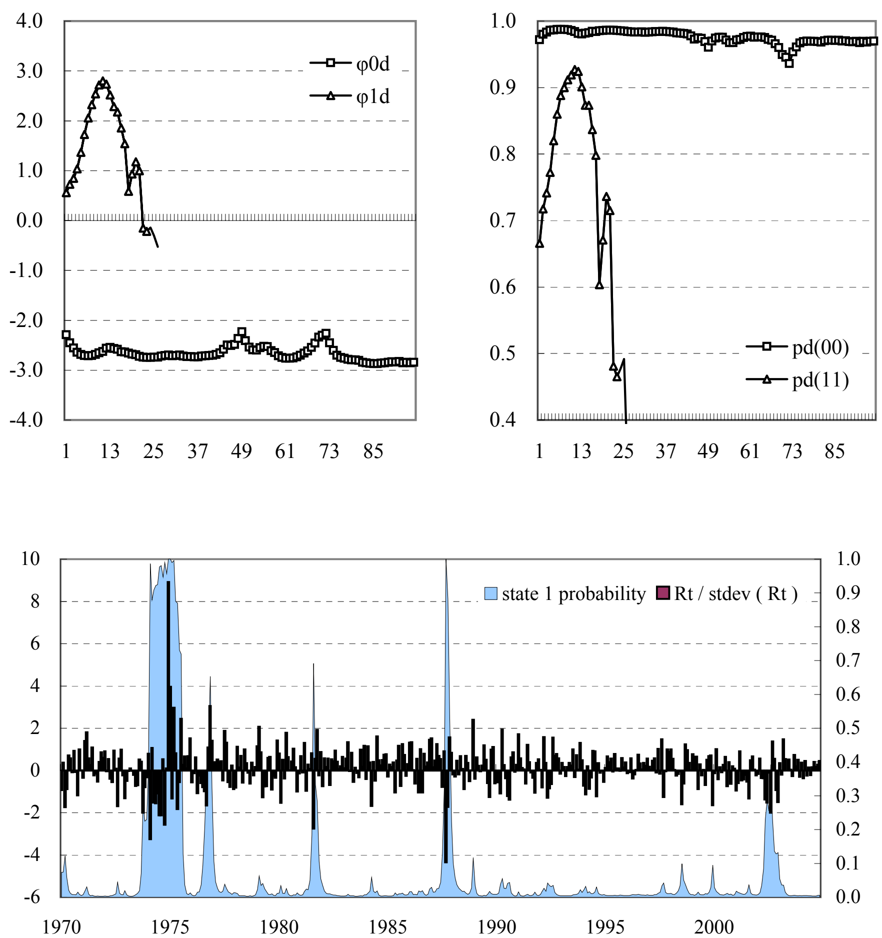

Figure 11: Bull-Bear and Duration Structure in France (1970:1-2004:12)

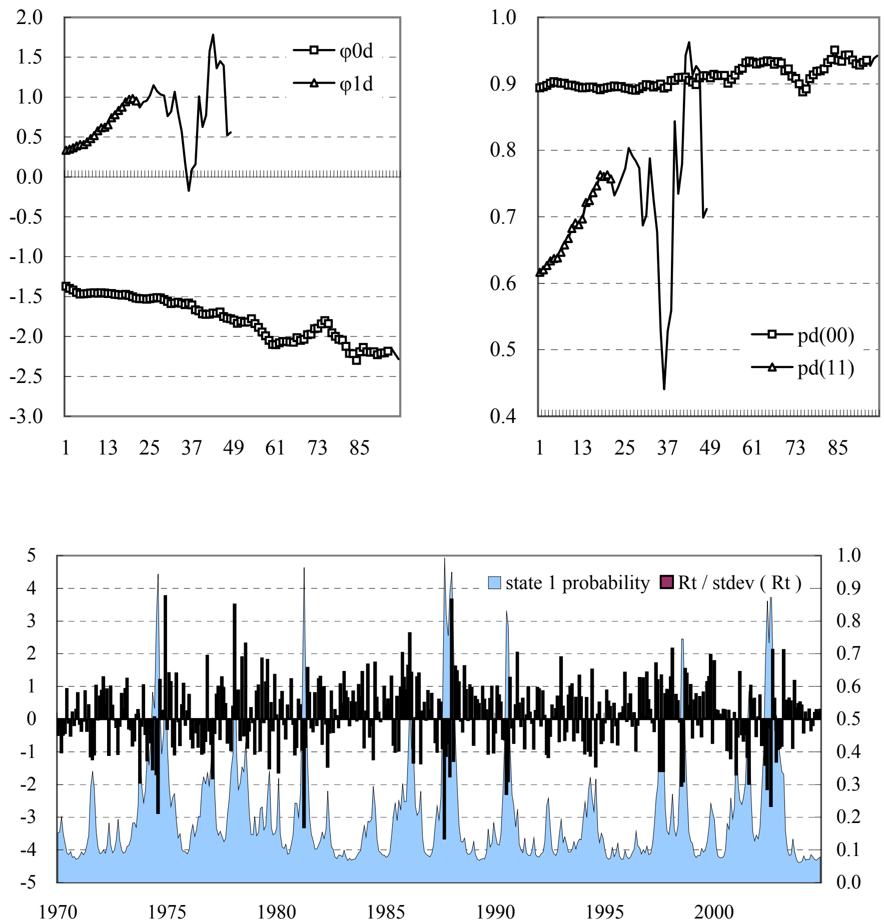

Figure 12: Bull-Bear and Duration Structure in Germany (1970:1-2004:12)

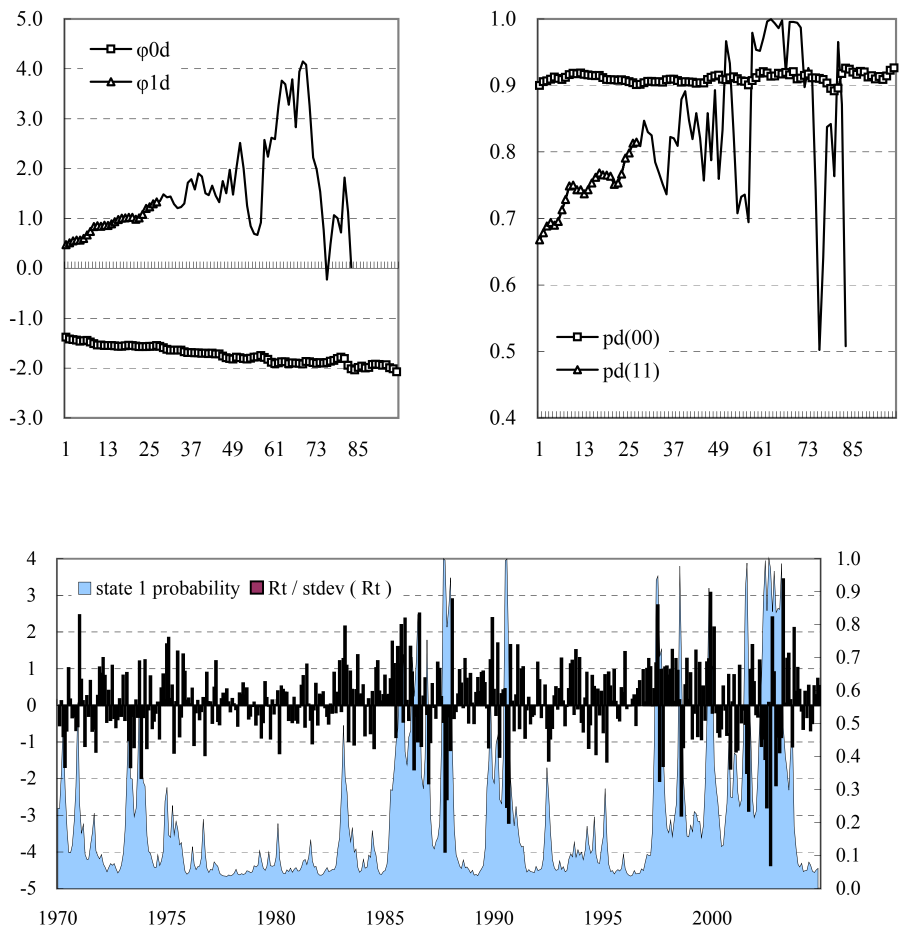

Figure 13: Bull-Bear and Duration Structure in Japan (1970:1-2004:12)

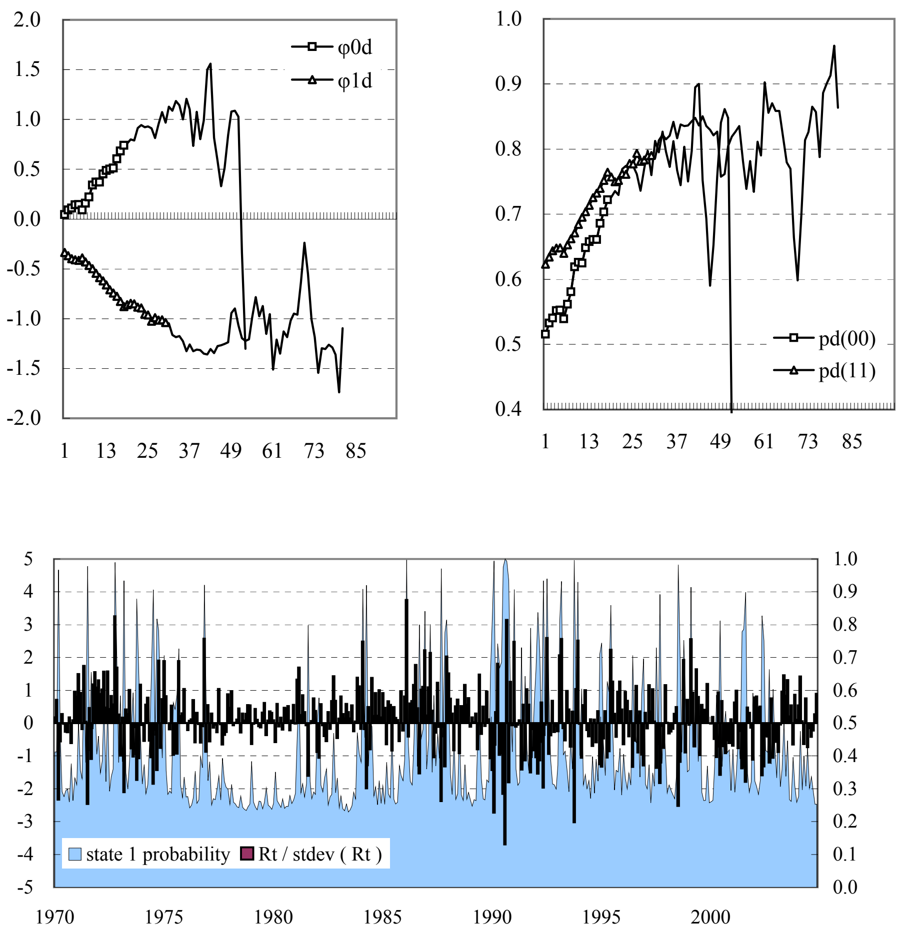

Figure 14: Bull-Bear and Duration Structure in Hong Kong (1970:1-2004:12)

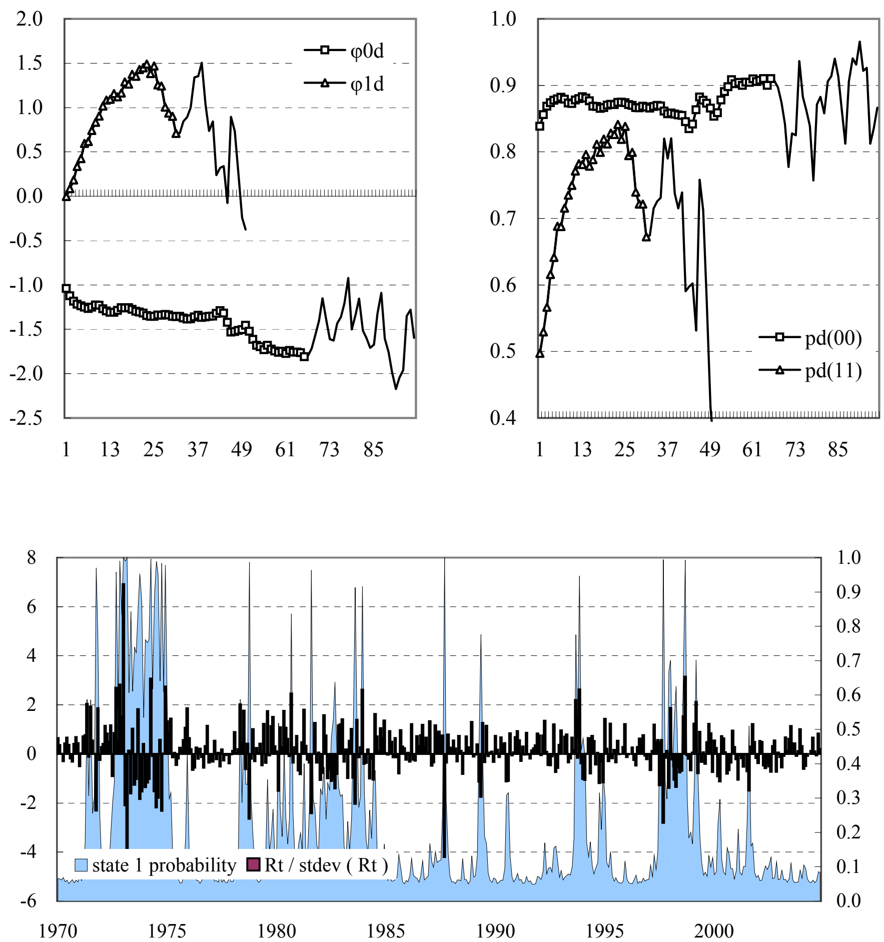

Figure 15: Bull-Bear and Duration Structure in Singapore (1970:1-2004:12)